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Local correlations in one- and two-dimensional disordered systems

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Abstract. In one-dimensional disordered systems all states are localized but when you add local correlations, such as a dimer impurity, some states become delocalized. In this paper, a wide range of correlations are studied leading to a delocalization of states. Moreover, second-nearest-neighbour models which also have extended states are investigated. Finally, a new approach to the case of two dimensions is presented, using a matrix-transfer technique, where explicit extended states are found.

1. Introduction

The localization properties of disordered systems were first examined by Anderson [1], who showed that certain states are localized due to disorder. His result was generalized by Mott and Twose [2] who proved that, in one dimension, all states are localized for any amount of disorder. This study was extended to higher dimensions by the ‘gang of four’ [3] using scaling properties. The conclusions of most existing studies on localization in two-dimensional disordered systems, as well as numerical studies [4], show that all states are also localized in two dimensions for any amount of disorder [5].

However, when local correlations are added to a disordered system, all states are no longer localized; sufficient extended states exist to alter the conduction properties. This was first shown by Flores [6] for the off-diagonal disordered discrete one-dimensional system. Very recently, Sil *et al* [7] found that all states are extended for a particular off-diagonal disordered model. A continuous model with delta impurities exhibiting delocalization for dimer-type correlations was studied by Sanchez *et al* [8]. Dunlap *et al* [9] worked out the extended states of the diagonal disordered system with dimer impurities. The diagonal dimer model was then extensively studied by Bovier [10] and by Flores and Hilke [11]. The model studied was a two-value random potential where the neighbour sites were equal by pairs. This model exhibits a localization length that diverges as $1/E^2$ around the delocalized state. This behaviour is not altered very much for states at the edges where a $1/E$ or a $1/\sqrt{E}$ dependence is found [9–11]. Evangelou and Economou [12] showed that the localization properties are essentially the same if, instead of considering a dimer impurity, larger clusters are examined with the main result being that the number of energies at which extended states exist is related to the size of the cluster.

In the following section we will study more general correlations. In particular, a model is presented where delocalized states exist for any type of disorder.

The third section is devoted to the study of a second-nearest-neighbour model. The main conclusion is that the localization properties of this model are essentially the same as those of

the nearest-neighbour model. In fact, by inducing specific correlations among the disordered sites, we calculate explicit extended wavefunctions. However, the divergence around the energy at which extended states exist has a different dependence on the localization length. The divergence is stronger than $1/E$, which may be interpreted as there being fewer extended states than in the nearest-neighbour model where the dependence can be $1/E^2$.

Finally, in section 4, we present a new approach to the case of two dimensions. We map the two-dimensional problem onto a one-dimensional discrete model in which we have nearest-neighbour hopping plus long-range hopping of the width of the system. If we correlate the disordered site energies in a similar way as for the dimer model, the system possesses completely extended states around a certain energy E_c . Away from this energy the localization length behaves as $1/\sqrt{E - E_c}$ and $1/|E - E_c|$.

2. Correlations in one dimension

Let us consider the disordered diagonal tight-binding model described by the following Hamiltonian

$$H = \sum_l V_l |l\rangle\langle l| + |l\rangle\langle l+1| + |l\rangle\langle l-1| \quad (1)$$

where $|l\rangle$ can be regarded as an atomic-like orbital centred at site $l \in Z$ and V_l is a random potential. To solve the Schrödinger equation $H|\psi\rangle = E|\psi\rangle$, we develop the wavefunction in a discrete space

$$|\psi\rangle = \sum \psi_l |l\rangle. \quad (2)$$

The Schrödinger equation can then be written as

$$\psi_{n+1} + \psi_{n-1} = (E - V_n)\psi_n \quad (3)$$

where V_n are random variables.

This equation can be written in a different form using the transfer-matrix technique.

$$\begin{pmatrix} \psi_{n+1} \\ \psi_n \end{pmatrix} = \begin{pmatrix} E - V_n & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \psi_n \\ \psi_{n-1} \end{pmatrix}. \quad (4)$$

Or, by redefining the variables

$$\Psi_{n+1} = T_n \cdot \Psi_n \quad (5)$$

where

$$\Psi_{n+1} = \begin{pmatrix} \psi_{n+1} \\ \psi_n \end{pmatrix} \quad \text{and} \quad T_n = \begin{pmatrix} E - V_n & -1 \\ 1 & 0 \end{pmatrix}. \quad (6)$$

It follows from equation (5) that

$$\Psi_{n+1} = \prod_{i=0}^n T_i \cdot \Psi_0. \quad (7)$$

Therefore, the relevant quantity to study is the product of these transfer matrices.

The simplest way to construct an extended wavefunction is to look at a product of transfer matrices. If one of the products gives an identity matrix, it will be easy to calculate the states. The simplest example of this is to take $E = V_0$, thus, $T_0^2 = -I$ where I is the identity matrix. Now, if one considers a two-value random number, for example $V_n = \pm V$, then for the energies $E = V$ or $E = -V$ the product of two identical transfer matrices equals $-I$. Let us now consider the following impurity model at $E = V$ with two different sites. The first is a pair site with $V_{n+1} = V_n = V$ which we will call site A and the second is a $V_n = W$ site called site B. Site A can be evaluated as the product of two transfer matrices with values $E - V = 0$ giving $-I$. Therefore, the A sites will not alter the localization properties of this model. On the other hand, the B sites will be determinant for these properties. Defining the localization length L_c as

$$L_c^{-1} = - \lim_{n \rightarrow \infty} \frac{1}{n} \ln |\psi_n| \tag{8}$$

shows that the localization length is mainly given by the logarithm of the eigenvalues of the product of the transfer matrices. As A sites do not contribute to the product, the localization length will be simply given by the absolute value of the inverse of the logarithm of the eigenvalues, as the eigenvalues of a product of identical matrices are simply the product of the eigenvalues. (The product of the two eigenvalues of one matrix is one, therefore their logarithm is equal in absolute values.) The eigenvalues of a 2×2 matrix can be expressed as

$$\text{Eig}(A) = \frac{1}{2} [\text{Tr}(A) \pm \sqrt{(\text{Tr}(A))^2 - 4 \text{Det}(A)}]. \tag{9}$$

As the determinant is always 1, the condition for the eigenvalues of site B being of modulus 1 is

$$|W - V| \leq 2. \tag{10}$$

If the corresponding transfer matrix of site B has eigenvalues of modulus 1, equation (7) immediately tells us that the state will be extended. This is probably the simplest example of a delocalized state due to a local correlation for a diagonal disorder. This model is almost equivalent to the first one proposed in [9] except that the B site is not a pair site. However, the localization properties are exactly the same.

We can now generalize this procedure to more sites. Let us call the sequence with the sites V_1, V_2, V_3, \dots site A and the one with the sequence W_1, W_2, W_3, \dots site B. To each of the two sites we associate the corresponding transfer matrix T_A and T_B , respectively. Hence,

$$T_A = \begin{pmatrix} E - V_1 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} E - V_2 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} E - V_3 & -1 \\ 1 & 0 \end{pmatrix} \dots \tag{11}$$

and equivalently for T_B . The simplest construction of extended states is to take $T_A = \pm I$ and $T_B = \pm I$ where I is the identity matrix. For two sites A we get the following conditions

$$E_c - V_1 = 0 \quad \text{and} \quad E_c - V_2 = 0. \tag{12}$$

E_c being the energy for which the states are extended. We can continue this procedure for different numbers of sites. However, the next interesting result comes from considering four sites where the conditions on the V_n s are obtained by calculating $T_A = -I$ and yield

$$E_c - V_1 = \frac{2}{E_c - V_2} \quad V_3 = V_1 \quad \text{and} \quad V_4 = V_2. \tag{13}$$

(13) shows that we can consider one random variable V_1 and correlate the other potentials to V_1 . The only constraint on V_1 is that it cannot be singular. V_2 is then defined as

$$V_2 = E_c - \frac{2}{E_c - V_1}. \quad (14)$$

For the ensuing discussion we will consider the simplest case where $E_c = 0$. The more general case $E_c \neq 0$ does not affect the conclusions but makes the discussion less transparent. The correlation (13) then becomes

$$V_2 = \frac{2}{V_1} \quad V_3 = V_1 \quad \text{and} \quad V_4 = \frac{2}{V_1}. \quad (15)$$

We can now consider *any* random distribution P of V_1 excluding 0, hence

$$P : V_1 \in \text{Re} - \{0\}. \quad (16)$$

In short, a disordered system with correlations between neighbouring sites, obeying (15), will have extended states for $E = 0$.

We will now study what happens around the energy $E = 0 + \epsilon$. In order to estimate the divergence of the localization length as a function of ϵ , we evaluate the eigenvalues λ of

$$T_A = \begin{pmatrix} \epsilon - V & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \epsilon - 2/V & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \epsilon - V & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \epsilon - 2/V & -1 \\ 1 & 0 \end{pmatrix}. \quad (17)$$

The result for the eigenvalues is the following:

$$\begin{aligned} \lambda_{\pm} = & -1 + \left(2 + \frac{2}{V^2} + \frac{V^2}{2}\right) \epsilon^2 - \left(\frac{2}{V} + V\right) \epsilon + \frac{1}{2} \epsilon^4 \pm \epsilon \left[-4 - \frac{4}{V^2} - V^2 + \left(2 + \frac{4}{V}\right) \epsilon \right. \\ & + \left(5 + \frac{4}{V^4} + \frac{8}{V^2} + 2V^2 + V^4/4\right) \epsilon^2 - \frac{8}{V^3} + \frac{12}{V} + 6V + V^3 \epsilon^3 \\ & \left. + \left(6 + \frac{6}{V^2} + 6V^2\right) \epsilon^4 - \left(\frac{2}{V} + V\right) \epsilon^5 + \frac{1}{4} \epsilon^6 \right]^{1/2}. \end{aligned} \quad (18)$$

If the expression inside the square root is negative then $|\lambda| = 1$, as the eigenvalues are complex conjugates of each other. On the other hand, if this expression is positive, which is essentially the case when $V > 1/\epsilon$ or $V < \epsilon$, we have, if $\epsilon/V \gg 1$

$$\lambda \sim \epsilon^2/V^2. \quad (19)$$

This would lead to a localization length depending on the energy like $(2 \ln(\epsilon/V))^{-1}$. As ϵ/V is large in this case, the localization length would be microscopic. In order to have a more interesting bound on this localization length, we apply one of Wittmeyer's [13] theorems for non-extreme values of V on eigenvalues resulting in the following statement:

$$|\text{Eig}(ABC\dots)|_{\max}^2 \leq (\text{Eig}(A^T A)_{\max})^{1/2} (\text{Eig}(B^T B)_{\max})^{1/2} (\text{Eig}(C^T C)_{\max})^{1/2} \dots \quad (20)$$

Evaluating the eigenvalues Λ_{\pm} of $T_A^T T_A$ to first order in ϵ , we obtain

$$\Lambda_{\pm} = 1 \pm \alpha \epsilon \quad (21)$$

where $\alpha = \sqrt{8 + 16/V^4 + 16/V^2 + 4V^2 + V^4}$. The product of Λ s is therefore an upper bound to the inverse of the localization length. Of course, the dependence of the localization length on ϵ will be influenced strongly by the distribution of the disordered site energies V . However, for a distribution in which V does not take values too close to 0 or ∞ , the localization length divergence would be bound by a $1/E$ dependence as given by equation (21). This result can be extended to the case $E_c \neq 0$ and using the definition (8) of the localization length L_c we have the bound

$$L_c \geq \left| \frac{\alpha^{-1}}{E - E_c} \right| \tag{22}$$

where α is evaluated at its maximum value.

In summary, in this section we have studied different kinds of correlations between neighbouring sites, generalizing the results of [9–12]. The main result is the study of a model admitting *any* type of disorder distribution which exhibits delocalization at a critical energy. The behaviour of the localization length near the critical energy depends strongly on the choice of the distribution but for a non-singular distribution we have a weaker divergence than $1/E$ which is similar to the dimer model where the divergence varies between $1/E^2$ and $1/E$ (inside the band). We have also studied models with clusters larger than four sites. Though the main features remain very similar, the number of free parameters differ. For clusters of size four we saw that we have one random variable fixing the other three variables. For clusters of size five and six we have two and three free parameters, respectively. But the next size of cluster, i.e. seven, has again only three free parameters. The relation between the number of free parameters and the size of the cluster is therefore non-trivial.

3. Second-nearest-neighbour model

In most cases, the tight-binding model relating a given system to a discrete Schrödinger equation supposes that only nearest-neighbour interactions are relevant. In the following, we study the effect of a second-nearest-neighbour hopping matrix element on the localization properties of a diagonal disordered system. For the sake of simplicity, we will consider the following second-nearest-neighbour interaction model:

$$\psi_{n+2} + \psi_{n+1} + \psi_{n-1} + \psi_{n-2} = (E - V_n)\psi_n. \tag{23}$$

This equation can be related to the following transfer-matrix formulation:

$$\begin{pmatrix} \psi_{n+2} \\ \psi_{n+1} \\ \psi_n \\ \psi_{n-1} \end{pmatrix} = \begin{pmatrix} -1 & E - V_n & -1 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \psi_{n+1} \\ \psi_n \\ \psi_{n-1} \\ \psi_{n-2} \end{pmatrix}. \tag{24}$$

Or, by redefining the variables

$$\Psi_{n+1} = T_n \cdot \Psi_n. \tag{25}$$

We can now study the eigenvalue properties of T_n . In fact, T_n admits four eigenvalues of modulus 1 if $-2 \leq E - V_n \leq 0$. For further study we will restrict ourselves to either

$E_c - V_n = 0$ or $E_c - V_m = -2$. The localization properties of the system will then be described by the product of the transfer matrices $\prod_n T_n$. We define site A as the one with $E_c - V_A = -2$ and site B the one with $E_c - V_B = 0$. By iterating the transfer matrices for clusters of the type AAA and BAB it turns out numerically that all eigenvalues of $\prod_n T_n$ are of modulus 1. This result can be understood analytically as one solution to the corresponding Schrödinger equation (22)

$$\begin{aligned}\psi_{3n+1} &= 0 \\ \psi_{3n+2} &= \pm \prod_j^n (V_{3n} - E - 1) \\ \psi_{3n+3} &= \mp \prod_j^n (V_{3n} - E - 1).\end{aligned}\tag{26}$$

This can be easily verified by inserting (26) into (23) and by noting that $E_c - V_{3n+1} = -2$ and $V - 3n + 2 = V_{3n}$. This solution for the cluster AAA|BAB is clearly overall extended.

Extended solutions to the disordered problem can also be obtained for other clusters; for example: ABA|BBB, ABB|BAB, BAB|BBA, ABB|BBA The common point to all these clusters is the fact that there is a long-range order in the sense that, for example, in the first cluster the $3n + 2$ site is always B. On the other hand, there also exist clusters with no long-range order, as ABBA|B and ABBA|BBB, for example, which do exhibit extended states.

In order to relate the degree of delocalization of the states to the model with only nearest-neighbour interaction, we evaluate the localization-length dependence on the energy. We will use the model corresponding to (26) which is based on clusters of the form AAA|BAB. The idea is to evaluate the product of transfer matrices when $E - V_A = -2 + \epsilon$ and $E - V_B = \epsilon$. The simplest sequence of impurities giving rise to eigenvalues of modulus different from 1 is BABAAAAA. We evaluate the eigenvalues to the first order in ϵ of the corresponding product of transfer matrices which is $T_B T_A T_B T_A T_A T_A T_A T_A T_A$. The eigenvalues λ of modulus closest to 1 are

$$\begin{aligned}\lambda &= 1 + \sqrt{6}\epsilon & \epsilon > 0 \\ \lambda^2 &= 1 - 3\epsilon & \epsilon < 0.\end{aligned}\tag{27}$$

As this is a particular sequence of site distribution leading to a divergence of the eigenvalues of an infinite product of transfer matrices, it would give a lower bound for this kind of divergence. Calculating the localization length with the help of equation (27), we obtain

$$L_c \leq \frac{K}{\sqrt{\epsilon}} \quad \epsilon > 0 \quad \text{and} \quad L_c \leq \frac{K'}{\epsilon} \quad \epsilon < 0\tag{28}$$

where K and K' are proportionality constants. The sequence studied here exhibits delocalized states if $E - V_A = -2$ and $E - V_B = 0$. This is actually not a general feature as the sequence ABB|BAB has delocalized states for $E_c - V_A = -v$ and $E_c - V_B = 0$ with $0 \leq v \leq 4$. This is a property which is equivalent to the dimer model with first-nearest-neighbour hopping except that $0 \leq v \leq 2$. The difference in the maximum value of v can be explained since the energy bandwidth of the non-disordered model is 4 in contrast to the second-nearest-neighbour model which has a bandwidth of 6.25.

The solution is the following

$$\begin{aligned} \psi_{3n} &= e^{ikn} & \text{where} & & v - 2 &= \cos k \\ \psi_{3n+1} &= -\psi_{3n} \\ \psi_{3n+2} &= (-1)^n \psi_2 + \beta_n. \end{aligned} \tag{29}$$

In order to verify that (29) is the solution, one has to take into account that we can rewrite $E_c - V_{3n} = -W_n$, $E_c - V_{3n+1} = W_n - v$ and $E - V_{3n+2} = 0$ with $W_n = 0$ or v . In addition

$$\beta_n = (-1)^n \left(\sum_j^{[n/2]} (e^{ik2j} W_{2j} - e^{ik(2j+1)} W_{2j+1}) \right) + \sum_j^n e^{ikj} 2i \sin k. \tag{30}$$

In the limit of $n \rightarrow \infty$, $\beta_n \rightarrow 0$. The solution (29) is therefore essentially independent of the disorder.

We also studied the behaviour of the localization length around E_c . As before, we define ϵ as $E - V_A = -v + \epsilon$ and $E - V_B = \epsilon$. The next step is to find a simple sequence leading to the eigenvalues all having a modulus different to 1. The sequence is, for instance, ABBBABBAB. We now evaluate the corresponding eigenvalues to the leading orders in ϵ . The result is essentially the same as for the previous case, i.e. (28), except for a single value of $v = 1$ where

$$L_c \leq \frac{K}{\epsilon^{1/4}} \quad \text{for} \quad \epsilon > 0. \tag{31}$$

This is actually the strongest non-exponential divergence observed for this type of model. We have shown in this section that extended states exist for a disordered system with second-nearest-neighbour interaction if local correlations between neighbouring sites exist. In our derivation we have only considered the case where the second-nearest-neighbour interaction is the same as the nearest-neighbour interaction. However, the results are essentially the same except that the transfer matrix T_n admits modular 1 eigenvalues for different values of $E - V_n$. We also derived the dependence in energy of the localization length beyond the extended states regime. The behaviour being stronger than a $1/E$, $1/\sqrt{E}$ or $1/E^{1/4}$ dependence.

In comparison to the dimer or polymer model [9, 12] of the next-nearest-neighbour model where the localization length diverges as $1/E^2$ for non-extreme values, the second-nearest-neighbour model exhibits a stronger localization, at least for the localization length. This does not actually imply that fewer states are extended as the only way to decide on this would be to compute the density of states near the critical energy and this task seems to be a difficult one.

4. Two-dimensional correlated disordered systems

In this section we develop a new approach to two-dimensional systems which is different from the usual transfer-matrix approach where one considers a system of width a [4]. This strip is then iterated to the desired length. The main inconvenience of this method is that one deals with very large non-trivial matrices. The size of the matrices are $2a \times 2a$ and each contains $2a$ random-site potentials. This approach is very successful for numerical calculations but not for analytical results.

We start with the following diagonal-disordered discrete Schrödinger equation

$$(V_{n,m} - E)\psi_{n,m} + \psi_{n,m+1} + \psi_{n,m-1} + \psi_{n+1,m} + \psi_{n-1,m} = 0. \quad (32)$$

We now consider a rectangular geometry of width a with the following periodic boundary conditions

$$\psi_{a+1,m} = \psi_{1,m+1} \quad \text{and} \quad \psi_{a,m} = \psi_{0,m+1}. \quad (33)$$

These boundary conditions correspond to periodic ones with a shift of one unit. In the continuum limit they are periodic. Equation (32) can then be rewritten as

$$(V_n - E)\phi_n + \phi_{n+a} + \phi_{n-a} + \phi_{n+1} + \phi_{n-1} = 0 \quad (34)$$

where

$$\phi_{am+n} = \psi_{n,m} \quad \text{and} \quad V_{n,m} \rightarrow V_{am+n}. \quad (35)$$

We could write equation (34) as a $2a \times 2a$ transfer matrix with one random-site potential per matrix but we can derive explicit extended states directly from the equation. We consider impurities of the form AB|BB where $E_c - V_A = v$ and $E_c - V_B = 0$ then (34) yields

$$\begin{aligned} (V_{2n} - E)\phi_{2n} + \phi_{2n+a} + \phi_{2n-a} + \phi_{2n+1} + \phi_{2n-1} &= 0 \\ \phi_{2n+1+a} + \phi_{2n+1-a} + \phi_{2n+2} + \phi_{2n} &= 0. \end{aligned} \quad (36)$$

We now consider only the case where a is odd and, by defining b as $2b + 1 = a$, equation (36) becomes

$$\begin{aligned} (V_{2n} - E)\phi_{2n} + \phi_{2(n+b)+1} + \phi_{2(n-b)-1} + \phi_{2n+1} + \phi_{2n-1} &= 0 \\ \phi_{2(n+b)+1} + \phi_{2(n-b)} + \phi_{2n+2} + \phi_{2n} &= 0. \end{aligned} \quad (37)$$

Therefore, the second part of equation (37) depends only on ϕ_{2n} and is independent of the disorder. This clearly shows that the states will be overall extended for any value of v with the simplest solution being

$$\phi_{2n} = 0 \quad \phi_{2n+1} = (-1)^{[2n+1/2b]}. \quad (38)$$

In the following we study this model for $E_c - V_A = v - \epsilon$ and $E_c - V_B = -\epsilon$. Equations (34) and (37) then yield

$$\begin{aligned} (V_{2n} - E_c + \epsilon)\phi_{2n} + \phi_{2(n+b)+1} + \phi_{2(n-b)-1} + \phi_{2n+1} + \phi_{2n-1} &= 0 \\ \epsilon\phi_{2n+1} + \phi_{2(n+b)} + \phi_{2(n-b)} + \phi_{2n+2} + \phi_{2n} &= 0. \end{aligned} \quad (39)$$

By defining

$$\Psi_n = \begin{pmatrix} \phi_{2n+2} \\ \phi_{2n+1} \end{pmatrix} \quad W_n = \begin{pmatrix} 1 & 0 \\ V_{2n} - E + \epsilon & 1 \end{pmatrix} \quad \text{and} \quad \epsilon_M = \begin{pmatrix} 1 & \epsilon \\ 0 & 1 \end{pmatrix}$$

we obtain from (39) that

$$W_n \Psi_n + \epsilon_M \Psi_{n-1} + \Psi_{n+a} + \Psi_{n-a-1} = 0. \tag{40}$$

Ψ_n can then be written as a sum of products of the type $W_n W_m \dots \Psi_{n_0}$, $\epsilon_M W_n \dots \Psi_{n_0}$ or $\epsilon_M \epsilon_M \dots \Psi_{n_0}$ where $0 \leq n_0 \leq 2a$. Thus, the only products contributing to the localization length are those whose eigenvalues are of modulus different from one. Hence, products which are of the form $\epsilon_M W_n$. Their corresponding eigenvalues are

$$\begin{aligned} \text{Eig}(\epsilon_M W_n) &= 1 + \sqrt{\epsilon V_n} & \epsilon V_n > 0 \\ \text{Eig}(\epsilon_M W_n) &= 1 + 2\epsilon V_n & \epsilon V_n < 0. \end{aligned} \tag{41}$$

In the case when $V_n > 0$ the localization length is given by

$$\begin{aligned} L_c &= 1/\sqrt{E - E_c} & (E - E_c) > 0 \\ L_c &= 1/2|E - E_c| & (E - E_c) < 0. \end{aligned} \tag{42}$$

5. Conclusions

Correlations between random sites can considerably alter the localization properties of disordered systems. The correlations presented in section 2 give rise to extended states for particular values of the energy, i.e. E_c . None of the correlations investigated lead to an energy band of delocalized states in the limit of infinite length. It would, however, be interesting to find out whether such a system exists. In the case of non-diagonal disorder Sil *et al* [7] found such a system. The model we studied most intensively is a cluster of four sites. The main new feature of this model is that it leads to delocalized states for *any* kind of random distribution contrary to models studied earlier which admitted an essentially two-value distribution. Each cluster is then parametrized by one random variable. If we equate the size L of the system with the localization length we can evaluate the energy bandwidth ΔE for which the states are overall extended. Using equation (22) we have

$$\Delta E \geq \alpha^{-1}/L. \tag{43}$$

α varies with the type of distribution chosen but is finite if the distribution avoids certain singular values such as E_c . Equation (43) clearly shows that the bandwidth drops to zero as a power law. This is very different to the Azbel [14] resonances where the bandwidth vanishes exponentially. The powerlaw decrease may, on the contrary, give rise to a non-zero conductivity as was first shown by Dunlap *et al* [9].

In section 3 the main question we tried to answer was what happens if one includes second-nearest-neighbour hopping. The main answer is that we also obtain extended states for particular values of the energy i.e. E_c . However, there are several differences. First, the correlations between the sites must have more constraints for the same size of clusters. We only found extended states for clusters larger than three. Second, the divergence of the localization length is always stronger than $1/E$ and we did not find any $1/E^2$ dependence. The strongest divergence found is actually a $1/E^{1/4}$ dependence. This seems to indicate that introducing second-nearest-neighbour hopping has a tendency to increase the localization

of the extended states. It would then be interesting to study the density of states in order to estimate the impact on the conduction properties.

Finally, in the fourth section we found explicit extended states for a two-dimensional disordered tight-binding model. The disorder we considered is one in which every second site is random. For this model we find essentially the same features as in the one-dimensional systems. The divergence of the localization length around the critical energy is of the type $1/E$ or $1/\sqrt{E}$ depending on the sign of the perturbation. We can point out that we could always calculate explicitly the wavefunction for the case where the energy was critical in the sense that the states are extended for this energy. The difficult task is to find the correlations leading to delocalization at a given energy and then to work out what happens around these energies. The localization length can often be estimated but other functions such as the conductivity or the density of states are more difficult to obtain. It would be very interesting to be able to calculate these functions also.

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